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16. Abstract

The optimization of descent maneuvers, calculated to expend the minimum amount of fuel. Certain limitations are taken into account--angle of re-entry, distance between the modules at the finish time, and direct visibility between the modules. It is easy to prove that the optimal descent orbits lie in the plane of the initial orbit.

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OPTIMIZATION OF THE DESCENT MANEUVERS OF A SATELLITE MODULE FROM PLANETARY ORBIT

S. N. Kirpichnikov

Let us consider a space vehicle moving along an elliptical orbit in the field of gravitation of a spherically symmetrical planet. The space vehicle consists of two parts: the recovery module (module I) and the orbital module (module II). It is necessary to construct a maneuver which is optimal in fuel consumption for the descent of module I to the planet. The maneuver is implemented with the use of a single initial pulse, applied to module I so that the orbital module continues to travel in the initial orbit, while the recovery module transfers to a trajectory of approach with the planet. The maneuver concludes with the landing of module I after re-entry into the dense atmospheric layers. The planet may be without atmosphere, but then the maneuver concludes with the hard touchdown of module I with its surface. In the latter case, we should everywhere equate the altitude of the atmosphere to zero. /5

The launch will imply the moment of the initial pulse. For purposes of definition, the finish will imply the landing of module I on the planet's surface. All the aforestated will, however, remain valid if we select some other fixed moment between the re-entry of the module I and the landing as our finish point. /6

The initial orbit of the space vehicle and the trajectory of the recovery module right until its re-entry are considered to be Keplerian; we will only examine elliptical descent orbits having motion which is straight with respect to the initial orbit. It is presumed that the earlier known angular range, altitude variation and time of motion in re-entry until the finish point are located

* Numbers in the margin indicate pagination in the foreign text.

in a planetocentric plane passing through the vector of velocity of module I during its re-entry.

With Tsiolkovsky's formula in mind, we will minimize the characteristic velocity of the initial pulse, i.e., the modulus of the pulse variation of velocity of module I.

In this study, the problem formulated is studied under complex limiting conditions. Notably incorporated are limitations on the angle of re-entry and the distance between modules at the moment of finish. Moreover, condition of direct visibility between modules or a more general limitation on the zenith distance of module II at the finish point is taken into account; limitation on the velocity of module I at the re-entry point may also be taken into the calculation.

Following the method proposed by Ting Lu [1], it is easy to prove that optimal descent orbits of these maneuvers, in all cases of interest, must lie in the plane of the initial orbit; therefore, this study is restricted to the coplanar statement of the problem.

1. Mathematical Statement of the Problem and General Conclusions

In the plane of motion, let us introduce polar coordinates r, θ, with origin at the center of the planet so the direction of positive reading of angle θ coincides with the direction of motion with respect to the initial orbit.

In addition to Keplerian orbital elements: semimajor axis a, eccentricity e and angular distance ω of the pericenter from the polar axis we will consider the elements p, q introduced by the formula

$$p=\frac{1}{\sqrt{a(1-e^2)}}, \quad q=\frac{e}{\sqrt{a(1-e^2)}}. \tag{1}$$

Let p_1, q_1, a_1, e_1, ω_1 be parameters of initial orbit; p, q, a, e, ω are parameters of the intermediate orbit of the descent module. /7

Let us designate the moment of time and the polar coordinates of the start and re-entry points of module I by t_1, r_1, ω_1 and t_2, r_2, ω_2, respectively. It is obvious that

$$r_2 = r_{pl} + h_{atm}, \tag{2}$$

where r_{pl}--radius of the planet, h_{atm}--altitude of dense layers of the atmosphere. Let us introduce the parameter p_2 in terms of the formula

$$p_2=\frac{1}{\sqrt{r_2}}. \tag{3}$$

Assumed that we are given change Δr of the polar radius, angular range $\Delta\theta$ and duration Δt of the flight of module I in the re-entry layers to the finish point. Therefore, time t_2 of the finish and the polar coordinates r_2, θ_2 of the finish point will be

$$\left.\begin{aligned}\tilde{r}_2&=r_2-\Delta r,\\ \tilde{\vartheta}_2&=\vartheta_2+\Delta\vartheta,\\ \tilde{t}_2&=t_2+\Delta t.\end{aligned}\right\} \tag{4}$$

If the finish implies the landing of module I on the surface of the planet, then

$$\Delta r=h_{\text{атм}}, \quad \tilde{r}_2=r_{\text{пл}}, \tag{5}$$

but if it is the module's re-entry, then

$$\Delta t=\Delta r=\Delta\vartheta=0, \quad \tilde{r}_2=r_2, \quad \tilde{\vartheta}_2=\vartheta_2, \quad \tilde{t}_2=t_2. \tag{6}$$

For polar radii r_1 and r_2 we find

$$r_1=[p_1^2+p_1 q_1 \cos(\vartheta_1-\omega_1)]^{-1}=[p^2+pq\cos(\vartheta_1-\omega)]^{-1}, \tag{7}$$

$$r_2=p_2^{-2}=[p^2+pq\cos(\vartheta_2-\omega)]^{-1}. \tag{8}$$

The characteristic velocity ΔU of the initial pulse can be reduced to the form [2]

$$\Delta U=K\Delta V, \tag{9}$$

$$\Delta V=\left\{q_1^2+3p_1^2+q^2-p^2+\frac{2p_1^3}{p}-2qq_1\cos(\omega_1-\omega)-\frac{2(p-p_1)^2 q_1}{p}\cos(\vartheta_1-\omega_1)\right\}^{\frac{1}{2}}, \tag{10}$$

where K--Gauss' constant multiplied by the square root of the planet's mass. For Δ V, differing only in the constant factor from ΔU, we retain the name of characteristic velocity. /8

The slope Φ_t of thrust during the initial pulse we will calculate in the opposite direction of the transversal to the direction of the pulse; then we will have

$$\operatorname{tg}\Phi_{\text{т}}=\frac{p\,[q\sin(\vartheta_1-\omega)-q_1\sin(\vartheta_1-\omega_1)]}{(p_1-p)\,[p_1+q_1\cos(\vartheta_1-\omega_1)]}. \tag{11}$$

The signs of the numerator and denominator on the right side of (11) coincide, respectively, with the signs of sin Φ_t and cos Φ_t.

Let us define the re-entry angle Φ as the angle between the velocity vector of the recovery module and the plane of the local horizon at time t_2:

$$\operatorname{tg}\Phi = -\frac{U_r}{U_\vartheta} = -\frac{pq\sin(\vartheta_2-\omega)}{p_2^2}, \quad \Phi \in \left[0, \frac{\pi}{2}\right), \tag{12}$$

where U_r and U_θ are, respectively, radial and transversal constituents of velocity $\bar{U}_{re}$ of module I at the moment it re-enters. The values U_r, U_θ, U_{re} are defined by the formulas

$$U_r = Kq\sin(\vartheta_2-\omega), \quad U_\vartheta = \frac{Kp_2^2}{p}, \tag{13}$$

$$U_{\text{вх}} = K\sqrt{2p_2^2+q^2-p^2}. \tag{14}$$

Given a moment in time $\tilde{t}_2$, the orbital module has a polar radius r_3 and polar angle θ_3. For the latter values we find that

$$\int_{\vartheta_1-\omega}^{\vartheta_2-\omega} \frac{dv}{p(p+q\cos v)^2} + K\Delta t = \int_{\vartheta_1-\omega_1}^{\vartheta_3-\omega_1} \frac{dv}{p_1(p_1+q_1\cos v)^2}. \tag{15}$$

$$r_3 = [p_1^2 + p_1 q_1 \cos(\vartheta_3-\omega_1)]^{-1}. \tag{16}$$

The distance l between modules at the moment of finish is equal to

$$l = [r_3^2 + \tilde{r}_2^2 - 2\tilde{r}_2 r_3 \cos(\vartheta_3 - \tilde{\vartheta}_2)]^{\frac{1}{2}}, \tag{17}$$

and the zenith distance z of module II at the finish point will be

$$z = \arccos\frac{r_3^2 - l^2 - \tilde{r}_2^2}{2\tilde{r}_2 l}. \tag{18}$$

These relationships show that we may take the following as the basic parameters defining descent maneuvers

$$p,\ q,\ \vartheta_1,\ \vartheta_2,\ \vartheta_3,\ \omega, \tag{19}$$

which are dependent and satisfy the following constraints: /9

$$\varphi_1 = p^2 + pq\cos(\vartheta_1-\omega) - p_1^2 - p_1 q_1 \cos(\vartheta_1-\omega_1) = 0, \quad (20)$$

$$\varphi_2 = p^2 + pq\cos(\vartheta_2-\omega) - p_2^2 = 0, \quad (21)$$

$$\varphi_3 = \int_{\vartheta_1-\omega}^{\vartheta_2-\omega} \frac{dv}{p(p+q\cos v)^2} + K\Delta t - \int_{\vartheta_1-\omega_1}^{\vartheta_2-\omega_1} \frac{dv}{p_1(p_1+q_1\cos v)^2} = 0. \quad (22)$$

The unknown optimal descent maneuver will imply a maneuver to which corresponds the lowest value of the characteristic velocity ΔV.

Let us move to consider additional limitations. Let us first examine only limitations on the variables l and Φ. The re-entry angle Φ must lie within some given interval:

$$\Phi_{min} \leq \Phi \leq \Phi_{max}. \quad (23)$$

The distance l between modules at the finish must not exceed a set maximum value L:

$$l \leqslant L. \quad (24)$$

After introducing the auxiliary substantial variables α, β, we will write conditions (23), (24) thus:

$$\varphi_4 = (\Phi_{min} - \Phi)(\Phi - \Phi_{max}) - \alpha^2 = 0, \quad (25)$$

$$\varphi_5 = l - L + \beta^2 = 0. \quad (26)$$

Therefore, under conditions (23), (24), the problem is mathematically reduced to minimization of function (10) in the set of substantial variables

$$p,\ q,\ \vartheta_1,\ \vartheta_2,\ \vartheta_3,\ \omega,\ \alpha,\ \beta, \tag{27}$$

which satisfy conditions (20)-(22), (25), (26). As we know, the derivatives with respect to all variables in (27) from the Lagrange function must be equal to zero

$$\Delta V+\sum_{i=1}^{5}\lambda_i\varphi_i, \tag{28}$$

where λ_1, λ_2, ..., λ_5 are unknown constants.

Analysis of the equation of the extremum corresponding to variable β is equal to zero and the condition $l = L$ is satisfied; or else the unknown solution corresponds to a relative minimum of function ΔV in an auxiliary problem in which conditions (24), (26) are admitted. Such systematic investigation is therefore recommended.

1. Investigate the problem without taking into account the limitations (23)-(26). Using the results of study [2] and a method analogous to that developed in article [3], we can easily show /10 that the sole stationary solutions in the particular case are those which are dervied in study [2] in the investigation of energetically optimal single-pulse flight between an elliptical initial and circular final orbit. These solutions are characterized by apsidal tangential conjunctions of the intermediate trajectories with the initial orbit and a circle of radius r_2. The trajectory emerging from the apocenter of the initial orbit always requires less fuel consumption versus a trajectory emerging from the pericenter. For both stationary solutions the equality

$$\Phi=0. \tag{29}$$

is fulfilled.

Point (29) generally lies beyond the interval of (23); we should thus turn our search to the relative minimums of function ΔV, retaining only conditions (20)-(22) and angle Φ. The analysis and solution of this problem for any fixed angle Φ is contained in study [3].

$$\Phi = \Phi_{min}. \quad (30)$$

If from all solutions derived at this stage there are those for which inequality (24) is satisfactory, then by comparing the characteristic velocities which correspond to them we find the desired solution. Otherwise, when condition (24) is violated in all solutions, the desired optimal maneuver is characterized by the equality

$$l = L, \quad (31)$$

and we should go on to point 2.

2. Investigate a problem in which are retained the conditions (20)-(22), (23), and inequality (24) is replaced by equality (31). For numerical study we can use the methods developed in the next section to define the optimal maneuver of descent for given values of l and Φ. The interval $[\Phi_{min}, \Phi_{max}]$ breaks into a series of points of equal parts and for each point calculation of the optimal maneuver is made. Then, by comparing functions ΔV, we find the approximate values of the angle Φ and the parameters of the desired optimal maneuver. If the accuracy obtained is insufficient, a localized specification of these approximate values may be carried out by one of the methods of successive approximations.

Let us now introduce, in addition to conditions (20)-(24), limitations on the variables z and U_{re}. The problem of optimization of function ΔV, just as in point 2, does not yield to analytic study. For its numerical solution we can use methods developed in the next section for constructing an optimal maneuver

of descent of module I for fixed parameters l and Φ, which allows /11
consideration of these limitations on the variables z and U_{re}. The definition of the desired solution is done as was stated above in point 2. The only difference is that here two intervals break down into a series of parts $[\Phi_{min}, \Phi_{max}]$, $[a_1(1 - e_1) - r_2, L]$, and calculation is made on all pairs of points, wherein one point is selected from the interval of variation of angle Φ, and the other from the interval of permissible values of distance l. This approach is especially convenient where great accuracy is not required, but it is important to produce a picture of change in the parameters of optimal maneuvers as a function of variations in quantities l and Φ.

2. Mathematical algorithm of constructing an optimal descent maneuver for fixed values of l and Φ.

Below is a description of the methods used to solve this problem under the assumption that are given the angle Φ of recovery module re-entry and the distance l between the modules at the moment of finish. Mathematically we must seek the smallest value of function (10) in the set of variables (19) which satisfy conditions (12), (17), (20)-(22), wherein parameters l and Φ are knowns. This method permits consideration also of limitations on variables z and U_{re}.

Let us consider that the initial orbit is not round, and the inequalities which follow are fulfilled

$$0 < q_1 < p_1, \tag{32}$$

$$r_{1p} > r_2, \tag{33}$$

$$r_{1p} < l + \tilde{r}_2, \tag{34}$$

$$r_{1a} > l - \tilde{r}_2, \tag{35}$$

where $r_{1p} = a_1(1 - e_1)$ and $r_{1a} = a_1(1 + e_1)$--distance of pericenter and apocenter of initial orbit, respectively. Condition

(33) signifies that the initial orbit is situated wholly outside the dense layers of the atmosphere.

If $r_{1a} < l - \tilde{r}_2$ or $r_{1p} > l + \tilde{r}_2$, then apparently the descent maneuvers with a given distance l are generally impossible. The cases of

$$\left.\begin{aligned} q_1&=0,\\ r_{1a}&=l-\tilde{r}_2,\\ r_{1p}&=l+\tilde{r}_2 \end{aligned}\right\} \tag{36}$$

will not be discussed here, since for each of them a selected problem loses its extremal nature and becomes determinant.

Let us now state the methods. From equalities (12), (21) we find that /12

$$q=\left[\frac{p_2^4\sec^2\Phi}{p^2}-2p_2^2+p^2\right]^{1/2}, \tag{37}$$

$$\operatorname{tg}(\vartheta_2-\omega)=\frac{-p_2^2\operatorname{tg}\Phi}{p_2^2-p^2}, \tag{38}$$

where the signs of the numerator and denominator in the last formula coincide with the signs $\sin(\theta_2 - \omega)$ and $\cos(\theta_2 - \omega)$, respectively.

Relationship (13) permits us to find that

$$\vartheta_2=\vartheta_3-\Delta\vartheta-\gamma_1\arccos\frac{r_3^2+\tilde{r}_2^2-l^2}{2r_3\tilde{r}_2},\quad \gamma_1=\pm1, \tag{39}$$

and from equality (20) we get

$$\left.\begin{aligned} \sin(\vartheta_1-\omega_1)&=\frac{-b_2b_3+\gamma_2b_1\sqrt{b_1^2+b_2^2-b_3^2}}{b_1^2+b_2^2},\\ \cos(\vartheta_1-\omega_1)&=\frac{-b_1b_3-\gamma_2b_2\sqrt{b_1^2+b_2^2-b_3^2}}{b_1^2+b_2^2},\\ \gamma_2&=\pm1, \end{aligned}\right\} \tag{40}$$

where

$$b_1 = pq\cos(\omega-\omega_1) - p_1 q_1, \quad b_2 = pq\sin(\omega-\omega_1), \quad b_3 = p^2 - p_1^2. \tag{41}$$

Henceforth, the symbols q, θ_1, θ_2, and ω will imply functions of the parameters p, θ_3, definable by relationships (37)-(41). With such exception of variables, the remaining unknowns p, θ_3 must satisfy the following inequalities:

$$0 < q < p, \tag{42}$$

$$|\tilde{r}_2^2 + r_3^2 - l^2| \leqslant 2r_3\tilde{r}_2, \tag{43}$$

$$b_1^2 + b_2^2 - b_3^2 \geqslant 0. \tag{44}$$

As a result of relationship (42), i.e., the supposition on the ellipticity of the transitional orbit, condition (22) is reduced to the form

$$\Delta = 0, \tag{45}$$

whereof

$$\Delta = K\Delta t + a^{\frac{3}{2}}[E_2 - E_1 - e(\sin E_2 - \sin E_1)] - a_1^{\frac{3}{2}}[E_2^{(1)} - E_1^{(1)} - e_1(\sin E_2^{(1)} - \sin E_1^{(1)})], \tag{46}$$

where E_1, E_2 and $E_1^{(1)}$, $E_2^{(1)}$ are eccentric anomalies of modules I and II at time t_1, t_2 respectively. Eccentric anomalies are easily calculated in terms of known formulas of elliptical motion. Time /13 $t_2 - t_1$ of motion before re-entry is calculated thus:

$$t_2 - t_1 = \frac{a^{\frac{3}{2}}}{K}[E_2 - E_1 - e(\sin E_2 - \sin E_1)]. \tag{47}$$

Let us now derive several inequalities and estimates which are employed in numerical solution. Above all, from condition (42) and the relationship

$$r_a \geqq r_{1p}, \tag{48}$$

where r_3--planetocentric distance of the apocenter of the transitional orbit, we find that

$$p_* < p \leqq p_{**}, \tag{49}$$

$$p_* = \frac{p_2 \sec \Phi}{\sqrt{2}}, \tag{50}$$

$$p_{**} = \max\left\{ \sqrt{\frac{p_2^4 \sec^2 \Phi - (p_1^2 + p_1 q_1)^2}{2\,[p_2^2 - p_1^2 - p_1 q_1]}},\quad \sqrt{p_1^2 + p_1 q_1} \right\}. \tag{51}$$

Let us note that here, the necessary and sufficient condition (44) of intersection of intermediate and initial orbits is replaced by necessary condition (48), thus henceforth in our solution we should take into account condition (48).

Relationship (43) and the apparent inequality

$$r_{1\text{п}} \leqslant r_3 \leqslant r_{1\text{а}} \tag{52}$$

produce

$$r_* \leqslant r_3 \leqslant r_{**}, \tag{53}$$

where

$$r_* = \max\{r_{1\text{п}},\ l - \tilde{r}_2\},\quad r_{**} = \min\{r_{1\text{а}},\ l + \tilde{r}_2\}, \tag{54}$$

$$r_{**} > r_*. \tag{55}$$

The latter inequality takes place due to relationships (34), (35).

Condition (53) defines two intervals, in which lie desired values of the angle θ_3:

$$\vartheta_3^{(1)} \leqslant \vartheta_3 \leqslant \vartheta_3^{(2)}, \tag{56}$$

$$2\pi - \vartheta_3^{(2)} \leqslant \vartheta_3 \leqslant 2\pi - \vartheta_3^{(1)}, \tag{57}$$

where

$$\left.\begin{array}{l} \vartheta_3^{(1)} = \omega_1 + \arccos \mu_1, \\ \vartheta_3^{(2)} = \omega_1 + \arccos \mu_2, \end{array}\right\} \tag{58}$$

$$\mu_1 = \frac{\frac{1}{r_*} - p_1^2}{p_1 q_1}, \qquad \mu_2 = \frac{\frac{1}{r_{**}} - p_1^2}{p_1 q_1}, \tag{59}$$

wherein it appears that /14

$$\mu_1 > \mu_2, \quad \mu_1 \leqslant 1, \quad \mu_2 \geqslant -1. \tag{60}$$

The methods developed permit taking into account limitations on the variables z and U_{re}. Let us introduce the requirement

$$z \leqslant z_{\max}, \tag{61}$$

where z_{max} is the prescribed miximum value of the zenith distance of module II at the finish point at time $\tilde{t}_2$. Then from formulas (18), (61) we yield

$$r_3 \geqslant r_{3*}, \tag{62}$$

$$r_{3*} = \sqrt{l^2 + \tilde{r}_2^2 + 2\tilde{r}_2 l \cos z_{\max}}. \tag{63}$$

Calculation of condition (62) now reduces to computation of the value of r_* in relationships (53)-(59) according to formula

$$r_* = \max\{r_{1u},\ l - \tilde{r}_2,\ r_{3*}\}. \tag{64}$$

In the particular case where limitation (61) is the condition of direct visibility between modules at the time of finish, the distance r_{3*} becomes equal to

$$r_{3*} = \sqrt{l^2 + \tilde{r}_2^2}. \tag{65}$$

Finally, the limitation on velocity U_{re} we will adopte in the form

$$U_{re} \leq U, \tag{66}$$

where the constant U is given. If we can ignore the velocity of rotation of the atmosphere versus the value U_{re}, then condition (66) is equivalent to the requirement that module I re-entry velocity does not exceed some fixed value of U.

From relationships (14), (37), (66), we find that

$$p \geqslant \tilde{p}, \quad \tilde{p} = \frac{K p_2^0 \sec \Phi}{U}, \quad \tilde{p} > p_*, \tag{67}$$

and consequently, condition (66) will be taken into account if inequality (49) is replaced by

$$\tilde{p} \leqslant p \leqslant p_{**}. \tag{68}$$

Henceforth, if one or both limitations (62), (66) is introduced, we will consider that the corresponding changes in inequalities (49), (53)-(59) have been made.

Let us formulate the derived results. The problem of optimization was reduced to searching for parameter p from the interval (49) and angle θ_3 in one of the regions (56), (57) so that conditions (44), (45), were fulfilled and function ΔV had its lowest /15

It is convenient in optimization to select the parameter p as a variable and the corresponding angle θ_3 to consider as lying in the region (56), (57) --- root of equation (45), for which conditions (44) is fulfilled.

According to the concrete selection of parameters γ_1, γ_2, four forms of maneuver must be studied. Let us introduce the quantity

$$j=\frac{5-2\gamma_1-\gamma_2}{2}, \quad \gamma_1=\pm 1, \quad \gamma_2=\pm 1, \tag{69}$$

which adopts the values j = 1, 2, 3, 4 for these forms. The descent trajectory for j = 1, 3 (γ_2 = 1) differs from the trajectory for j = 2, 4 (γ_2 = -1) by the concrete selection of the launch point in one of two points of intersection of the initial and intermediate orbits. Thus, in one case, the flight trajectory of module I will go beyond the initial area limited by the initial orbit; in the other case, it will be totally within this region. Furthermore, maneuvers j = 1,2 (γ_1 = 1) differ from maneuvers j = 3, 4 (γ_1 = -1) in the fact that at the moment of finish, in the first instance, the orbital module overtakes the descent module, i.e., it has a greater polar angle. In the second, however, the polar angle of the descent module is greater than the polar angle of the orbital module. Let us note that with this comparison, angles $\tilde{\theta}_2$, $\tilde{\theta}_3$ should be brought to the interval $|\tilde{\theta}_2 - \theta_3| \leq \pi$.

Let us cite a brief description of a computer program which employs the above algorithm.

I. A search is made of the approximate unknown value of the parameter p by a global scanning of interval (49), which is broken into n equal parts. For each point of division, all roots of equation (45) are found which lie in areas (56), (57) and for which condition (44) is fulfilled. In calculating the roots, rough constant accuracies are taken, which ensures the rapidity of operation of this unit of the program. Of the multitude of all selected values of parameter p and its corresponding roots, the pair p, θ_3 are chosen to which corresponds the smallest value of function ΔV. These values are taken as approximate optimal values.

II. The precise optimal value of variable p and other parameters of the desired maneuver are found. Let us note that the value ΔV as a function of parameter p for optimal selection of root θ_3 can have discontinuities of the first order and areas of nonexistence, where in equation (45) there are generally no roots of the necessary type. Due to the particular nature of the problem, the specification of optimal values of the parameters to the production of a prescribed accuracy should be done by the method of successive approximations. In each approximation, the interval between two values of the parameter p, adjacent to the optimal value of the /16 preceeding approximation, is broken down into some prescribed number of parts and calculation is carried out for all points of division. By comparison of the corresponding characteristic velocities we determine the optimal value of the parameters of a given approximation. For calculation of the roots of function Δ, we take constants ensuring the prescribed accuracy of computations.

In conclusion, let us touch upon the procedure of calculating roots of function (46) for any fixed value of parameter p. For determinacy, we will limit ourselves to descent maneuvers, whose total time $t_2 - t_1$ of maneuver is strictly less than the period of motion along the initial orbit. Accordingly, we will calculate eccentric anomalies appearing in equality (45)-(47), bearing in mind the condition

$$\left.\begin{array}{c} E_1+2\pi > E_2 \geqslant E_1, \\ E_1^{(1)}+2\pi \geqslant E_1^{(1)}+2\pi-\delta \geqslant E_2^{(1)} \geqslant E_1^{(1)}, \end{array}\right\} \quad (70)$$

where δ is any prescribed small quantity.

Areas (56), (57) are broken into N equal parts and for all points of division the function Δ is calculated in succession.

Let us point out one aspect of this method. For any value of the angle θ_3, we first determine the geometric picture of the maneuver, i.e., the juxtaposition of orbits, and then select specific branches of the modules' flight along orbits from the condition of fulfillment of inequalities (70). Thus, the critical values of angle θ_3 can exist, to which corresponds the discontinuity of function $E_2 - E_1$, if the flight branches in the descent orbit are switched; and of function $E_2^{(1)} - E_1^{(1)}$, if the switching took place on the trajectory of travel of module II. At points of switching, one of the maximum values of the corresponding functions is equal to zero, and the magnitude of the discontinuity is equal to 2π. It is easy to see that the roots of function Δ may lie only at a finite distance from the critical points. Let us select the number N so that the range of functions $E_2 - E_1$, $E_2^{(1)} - E_1^{(1)}$ in each segment of fine subdivision of intervals (56), (57) does not exceed some constant A. The range of functions in segments containing critical points will then be at least $2\pi - A$. By increasing N and selecting a constant A, we can always fulfill condition $A < 2\pi - A$ and thereby produce a criterion which will permit us, by the magnitude of the range of function Δ to judge whether or not switching occurred within the segment.

Let us return to a description of the procedure of calculating roots. If at the boundaries of some segment of division of intervals (56), (57) function Δ has different signs, and its range is less than the constant A, then within this segment is found the root of function Δ , which is calculated using a series of interpolations.

Let us pause on another aspect of the algorithm. If for any value of θ_3 the condition (44) is violated, then we will say that this value is in the area of non-existence of function Δ. When, in selecting points from intervals (56), (57) we enter the range of non-existence of function Δ, then with the next division of the given segment we suddenly find the boundary of the area of non-existence. In fulfilling the conditions of the presence of a root in this area, the root is calculated by a series of interpolations. A similar procedure is carried out when functions Δ move from the area of nonexistence to values of angle θ_3 for which condition (44) is fulfilled. /17

After we have found all roots of function Δ for the parameter p, we select a root from them to which corresponds a smaller value of function ΔV. With this concludes the determination of roots of function Δ.

3. Numerical Example

As an example of the algorithm developed in section 2, let us consider the problem of sending from the space vehicle--the sputnik Venus moving on an elliptical orbit--a probe (module I) to study the upper layers of the atmosphere of Venus. Let us minimize the fuel expenditure.

Let us choose the following elements of the initial orbit of the space vehicle:

$$a_1 = 10\,000 \text{ км}, \quad e_1 = 0{,}28, \quad \omega_1 = 0°. \tag{71}$$

The height of the pericenter of the orbit above Venus' surface will be 1,000 kilometers, while the height of the apocenter--6,600 km.

For numerical values of constants K and r_2, let us use

$$\boxed{\begin{aligned} K^2 &= 3{,}2423 \cdot 10^5 \ \text{км}^3/\text{см}^2, \\ r_2 &= 6200 \ \text{км}. \end{aligned}} \tag{72}$$

Let us assume further that the moment of finish coincides with the moment the probe enters the dense layers of atmosphere, then

$$\boxed{\Delta r_2 = \Delta t = \Delta \vartheta = 0.} \tag{73}$$

Let us require that at the moment of finish, the values l and $\Phi^{\cdot}$ have prescribed values and are fulfilled the conditions of direct visibility between the modules.

The calculations were performed on a M-20 computer. Two distance values were selected

$$l = 1{,}200 \text{ km}, \qquad l = 1{,}600 \text{ km} \tag{74}$$

and the interval of variation of angle Φ with spacing of 5° from zero to 30° was considered. The results are given in Table 1 and 2. Let us note that for both distance values l, the minimum characteristic velocity as a function of the angle Φ is attained for several values of the angle $\Phi \in (0°, 5)$, while this velocity rapidly increases for $\Phi > 10°$. It is also of interest to note that the velocity U_{re} for angles Φ not exceeding 10-15% in optimal maneuvers virtually is independent of angle Φ and is equal to roughly 8.3 km/s.

TABLE 1. PARAMETERS OF OPTIMAL MANEUVERS FOR 1 = 1,200 km

Φ	j	a, km	e	θ_l	θ_l	θ_l	ω	ΔU, km/s	Φ_T	t_2-t_l, s.	z	U_{re}, km/s
0°	3	9497.4	0.3472	184°99	368°61	2°95	8°61	0.348	126°14	5305	36°03	8.394
5	3	9170.7	0.3343	222°83	355°63	350°25	15°74	0.329	149°09	3259	34°32	8.321
10	3	8949.2	0.3488	228°52	339°73	336°40	19°59	0.403	156°54	2787	20°80	8.268
15	1	8394.2	0.3616	245°54	330°91	331°27	31°62	0.601	163°91	2055	2°21	8.122
20	2	7486.4	0.3782	281°72	331°32	333°69	56°05	1.080	195°98	1045	14°73	7.828
25	2	7000.1	0.4351	301°31	332°70	335°88	73°93	1.773	214°39	645	19°83	7.634
30	2	6760.9	0.5051	311°70	334°02	337°69	85°85	2.480	222°28	468	22°96	7.526

TABLE 2. PARAMETERS OF OPTIMAL MANEUVERS FOR 1 = 1,600 km

Φ	j	a, km	e	θ_l	θ_l	θ_l	ω	ΔU, km/s	Φ_T	t_2-t_l, s.	z	U_{re}, km/s
0°	3	9298.9	0.3333	219°64	372°46	361°74	12°46	0.268	156°68	3589	56°84	8.350
5	3	9321.2	0.3448	214°01	350°65	340°66	10°29	0.261	167°25	3491	52°26	8.355
10	3	9172.4	0.3633	212°90	333°36	325°39	11°92	0.328	167°40	3276	40°44	8.321
15	3	8888.3	0.3903	216°58	319°84	315°04	16°38	0.453	166°52	2918	23°71	8.253
20	1	8459.2	0.4242	222°85	310°42	310°42	24°15	0.655	162°30	2553	1°02	8.140
25	2	7698.6	0.4580	249°41	309°20	312°42	41°54	1.003	180°46	1580	15°80	7.904
30	2	7026.9	0.5103	270°74	310°54	315°59	59°02	1.552	197°20	999	24°99	7.645

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